

# D=6 massive spinning particle

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## Abstract

The massive spinning particle in six-dimensional Minkowski space is described as a mechanical system with the configuration space  $R^{5,1} \times CP^3$ . The action functional of the model is unambiguously determined by the requirement of identical (off-shell) conservation for the phase-space counterparts of three Casimir operators of Poincaré group. The model is shown to be exactly solvable. Canonical quantization of the model leads to the equations on wave functions which prove to be equivalent to the relativistic wave equations for the irreducible  $6d$  fields.

## 1 Introduction

The classical description of the relativistic spinning particles is one of traditional branches of theoretical physics having a long story [1, 2, 3, 4]. By now the several approaches to this problem have been developed. Most of them are based on the enlargement of the Minkowski space by extra variables, anticommuting [5, 6, 7, 8, 9] or commuting [10, 11, 12, 13, 16], responsible for the spin evolution.

In the recent paper [13], the new model was proposed for a massive particle of arbitrary spin in  $d = 4$  Minkowski space to be a mechanical system with the configuration space  $R^{3,1} \times S^2$ , where two sphere  $S^2$  corresponds to the spinning degrees of freedom. It was shown that principles underlying the model have simple physical and geometrical origin. Quantization of the model leads to the unitary massive representations of the Poincaré group. The model allows the direct extension to the case of higher superspin superparticle [14] and the generalization to the anti-de Sitter space [15].

Despite the apparent simplicity of model's construction its higher dimensional generalization is not so evident, and the most crucial point is the choice of configuration space for spin. In this talk we describe the massive spinning particle in six-dimensional Minkowski space  $R^{5,1}$ , that may be considered as a first step towards the uniform model construction for all higher dimensions. It should be also noted that this generalization may have a certain interest in its own rights since six is the one in every four remarkable dimensions: 3, 4, 6 and 10 where the classical theory of Green-Schwarz superstring can be formulated.

Let us now sketch the broad outlines of the construction. First of all, for any even dimension  $d$ , the model's configuration space is chosen to be the direct product of Minkowski space  $R^{d-1,1}$  and some  $m$ -dimensional compact manifold  $K^m$  being a homogeneous transformation space for the Lorentz group  $SO(d-1,1)$ . Then the manifold  $M^{d+m} = R^{d-1,1} \times K^m$  proves to be the

extended phase space of the model. It is well-known that the massive unitary irreducible representations of the Poincaré group are uniquely characterized by the eigenvalues of  $d/2$  Casimir operators

$$C_1 = \mathbf{P}^2, \quad C_{i+1} = \mathbf{W}^{A_1 \dots A_{2i-1}} \mathbf{W}_{A_1 \dots A_{2i-1}}, \quad i = 1, \dots, \frac{d-2}{2},$$

where  $\mathbf{W}_{A_1 \dots A_{2i-1}} = \epsilon_{A_1 \dots A_d} \mathbf{J}^{A_{2i} A_{2i+1}} \dots \mathbf{J}^{A_{d-2} A_{d-1}} \mathbf{P}^{A_d}$  and  $\mathbf{J}_{AB}, \mathbf{P}_C$  are the Poincaré generators. This leads us to require the identical (off-shell) conservation for the quantum numbers associated with the phase space counterparts of Casimir operators. In other words  $d/2$  first-class constraints should appear in the theory.

Finally, the dimensionality  $m$  of the manifold  $K^m$  is specified from the condition that the reduced (physical) phase space of the model should be a homogeneous symplectic manifold of Poincaré group (in fact it should coincide with coadjoint orbit of maximal dimension  $d^2/2$ ). The simple calculation leads to  $m = d(d-2)/4$ . In the case of four-dimensional Minkowski space this yields  $m = 2$  and two-sphere  $S^2$  turns out to be the unique candidate for the internal space of the spinning degrees of freedom. In the case considered in this paper  $d = 6$ , and hence  $m = 6$ . As will be shown below the suggestive choice for  $K^6$  is the complex projective space  $CP^3$ .

The models can be covariantly quantized à la Dirac by imposing the first-class constraints on the physical states being the smooth complex functions on the homogeneous space  $M^{d(d+2)/4} = R^{d-1,1} \times K^{d(d-2)/4}$

$$(\hat{C}_i - \delta_i) \Psi = 0, \quad i = 1, \dots, \frac{d}{2},$$

where the parameters  $\delta_i$  are the quantum numbers characterizing the massive unitary representation of the Poincaré group. Thus the quantization of the spinning particle theories reduces to the standard mathematical problem of harmonic analysis on homogeneous spaces. It should be remarked that manifold  $M^{d(d+2)/4}$  may be thought of as the *minimal* (in sense of its dimensionality) one admitting a non-trivial dynamics of arbitrary spin, and hence it is natural to expect that the corresponding Hilbert space of physical states will carry the *irreducible* representation of the Poincaré group.

Our spinor notations and conventions mainly coincide with those adopted in ref. [17] except for inverse signature of the Lorentz metric.

## 2 Classical theory

We start to construct the model with describing of the covariant parametrization for the spinning sector of the configuration space chosen as  $CP^3$ . It is useful to realize  $CP^3$  as complex projective space parametrized by the four-component left-handed Weyl spinor  $\lambda_a$ ,  $a = 1, \dots, 4$  subject to the equivalence relation  $\lambda_a \sim \alpha \lambda_a$ ,  $\alpha \in C \setminus \{0\}$ . Since the Lorentz transformations of the spinors obviously commute with the projective ones generated by the vector fields

$$d = \lambda_a \partial^a, \quad \bar{d} = \bar{\lambda}_a \bar{\partial}^a \quad (1)$$

the action of the Lorentz group can thereby be transferred from  $C^4$  to  $CP^3$ . (In rel. (1)  $\bar{\lambda}_a = B_a^{\dot{a}} \bar{\lambda}_{\dot{a}}$  and  $B$  is the Lorentz invariant matrix converting the dotted indices into the undotted ones [17].) The action of the Poincaré group on  $M^{12} = R^{5,1} \times CP^3$  is generated by the vector fields

$$P_A = \partial_A, \quad J_{AB} = x_A \partial_B - x_B \partial_A + ((\sigma_{AB})_a^{\phantom{a}b} \lambda_b \partial^a + c.c.), \quad (2)$$

To construct the Lagrangian, we consider all the possible Poincaré invariants of the particle's world-line on  $M^{12}$ . There are only three first-order invariants<sup>1</sup>

$$s = (\dot{x}^2)^{1/2} \quad , \quad \xi = \frac{(\dot{\lambda}_a \dot{x}^{ab} \lambda_b)(\dot{\bar{\lambda}}_c \dot{x}^{cd} \bar{\lambda}_d)}{\dot{x}^2 (\bar{\lambda}_a \dot{x}^{ab} \lambda_b)^2} \quad , \quad \eta = \frac{\epsilon^{abcd} \dot{\lambda}_a \bar{\lambda}_b \dot{\bar{\lambda}}_c \lambda_d}{(\bar{\lambda}_a \dot{x}^{ab} \lambda_b)^2} \quad , \quad (3)$$

where  $\epsilon^{abcd}$  is the Lorentz invariant spin-tensor [17], totally antisymmetric in its indices. Then the most general Poincaré and reparametrization invariant Lagrangian looks like:  $\mathcal{L} = sF(\xi, \eta)$ , with  $F$  being arbitrary function. To specify the particular form of the Lagrangian we require the presence of three local symmetries corresponding to the off-shell conservation of the Nöether charges associated with the classical counterparts of Casimir operators

$$C_1 = P^2 + m^2 \equiv 0 \quad (4)$$

$$C_2 = W_{ABC}W^{ABC} - m^2(\delta_1^2 + \delta_2^2) \equiv 0, \quad C_3 = W_A W^A + m^2\delta_1^2\delta_2^2 \equiv 0$$

Here  $W^A = \epsilon^{ABCDE} J_{BC} J_{DE} P_F$ ,  $W^{ABC} = \epsilon^{ABCDE} J_{DE} P_F$  are Pauli-Lubanski vector and tensor respectively;  $P_A = p_A$  and  $J_{AB} = x_{AP} x_B - x_{BP} x_A + ((\sigma_{AB})_a{}^b \lambda_b \pi^a + c.c.)$  are the Nöether charges associated with the global Poincaré invariance of the theory and, finally,

$$p_A = \frac{\partial \mathcal{L}}{\partial \dot{x}^A} \quad , \quad \pi^a = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}_a} \quad (5)$$

are the canonical momenta. The parameter  $m$  entering rels. (4) is nothing but the mass of the particle while the parameters  $\delta_1$  and  $\delta_2$  relate to the particle's spin. The substitution of the explicit expressions for the momenta (5) into the conditions (4) yields the set of differential equations for  $F$ , resolving which we come to the following Lagrangian:

$$\mathcal{L} = \sqrt{-\dot{x}^2 \left( m^2 - 4\delta_1^2 \frac{\epsilon^{abcd} \dot{\lambda}_a \bar{\lambda}_b \dot{\bar{\lambda}}_c \lambda_d}{(\bar{\lambda}_a \dot{x}^{ab} \lambda_b)^2} + 4m \sqrt{(\delta_2^2 - \delta_1^2) \frac{\epsilon^{abcd} \dot{\lambda}_a \bar{\lambda}_b \dot{\bar{\lambda}}_c \lambda_d}{(\bar{\lambda}_a \dot{x}^{ab} \lambda_b)^2}} \right)} + 2\delta_1 \left| \frac{\dot{\lambda}_a \dot{x}^{ab} \lambda_b}{\bar{\lambda}_a \dot{x}^{ab} \lambda_b} \right| \quad (6)$$

The Lagrangian (6) is obviously Poincaré invariant and possesses, by construction, five gauge symmetries two of which are the local  $\lambda$ -rescalings:  $\lambda_a \sim \alpha \lambda_a$  and one may be associated with the reparametrizations of the particle's world-line.

In the Hamiltonian formalism the model is completely characterized by the set of five first-class constraints three of which are dynamical

$$T_1 = p^2 + m^2 \approx 0, \quad T_2 = \bar{\lambda}_a p^{ab} \lambda_b \bar{\pi}^c p_{cd} \pi^d + m^2 \delta_1^2 \approx 0, \quad T_3 = \lambda_a \bar{\lambda}_b \pi^a \bar{\pi}^b + \delta_2^2 \approx 0 \quad (7)$$

and the other two are kinematical

$$T_4 = \pi^a \lambda_a \approx 0 \quad , \quad T_5 = \bar{\pi}^a \bar{\lambda}_a \approx 0 \quad (8)$$

generating the  $\lambda$ -rescalings with respect to the canonical Poisson brackets

$$\{x^A, p_B\} = \delta^A_B, \quad \{\lambda_a, \pi^b\} = \delta^b_a, \quad \{\bar{\lambda}_a, \bar{\pi}^b\} = \delta^b_a \quad (9)$$

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<sup>1</sup>Here we left out the invariants transforming by a total derivative under the projective transformations which

The corresponding first-order (Hamiltonian) action associated with the constraints (7), (8) looks like

$$S_H = \int d\tau \left\{ p_A \dot{x}^A + \pi^a \dot{\lambda}_a + \bar{\pi}^a \dot{\bar{\lambda}}_a - \sum_{i=1}^5 e_i T^i \right\} , \quad (10)$$

where  $e_i$  are the Lagrange multipliers to the constraints, and  $e_4 = \bar{e}_5$ . Notice that despite the quite nonlinear structure of the constraints, the classical equations of motion are completely integrable for the action (10) with the arbitrary Lagrange multipliers  $e_i$ . This fact is not surprising as the model, by construction, describes a free relativistic particle possessing sufficient number of symmetries. Here, however, we omit the explicit expressions for the Hamiltonian equations as well as its solution in the spinning sector. (The more detailed treatment of this subject will be given in ref. [19]) In the Minkowski space the corresponding solution reads

$$\begin{aligned} x^A(\tau) &= x_0^A + 2(E_1 + E_2 \delta_1^2) p^A - m^{-2} V^A(\tau) \\ V^A(\tau) &= V_1^A \cos(2m^2 E_2 \delta_1) + V_2^A \sin(2m^2 E_2 \delta_1) \end{aligned} \quad (11)$$

Here  $E_i(\tau) = \int_0^\tau d\tau e_i(\tau)$ , the constant vector of the six-momentum  $p_A$  is assumed to be chosen on the mass shell and the constant vectors  $V_1, V_2$ , being defined by initial data for the spinning degrees of freedom, are constrained to satisfy

$$p_A V_{1,2}^A = 0 , \quad V_1^2 = V_2^2 = \delta_1^2 - \delta_2^2 , \quad (V_1, V_2) = 0 \quad (12)$$

As is seen from (11) the space-time evolution is completely determined by the independent evolution of the two Lagrange multipliers  $e_1, e_3$ . The presence of the additional gauge invariance in the solutions (11), as compared with the spinless particle case, causes the conventional notion of the particle's world-line to fail. Instead, according to (11), one has to consider the classes of gauge equivalent trajectories which in the case under consideration are identified with the two-dimensional tubes of radius  $\rho = \sqrt{\delta_1^2 - \delta_2^2}$  along the particle's momenta  $p_A$ . So, in each moment of time (which may be chosen by imposing the gauge fixing condition  $x^0 = c\tau$ ) the massive spinning particle is not localized in a certain point of Minkowski space but represents a string-like configuration contracting to a point only provided that  $\delta_1 = \delta_2$ .

Finally, let us discuss the structure of the physical observables of the theory. Each physical observable  $A$  being a gauge-invariant function on the phase space should meet the requirements:

$$\{A, T_i\} = 0 \quad , \quad i, j = 1, \dots, 5 \quad (13)$$

Due to the obvious Poincaré invariance of the constraint surface, the generators  $J_{AB}, P_C$  automatically satisfy (13) and thereby are observable. On the other hand, it is easy to compute that the dimensionality equals 18 of the physical phase space of the theory. Thus the physical subspace may covariantly be parametrized by 21 Poincaré generator subject to 3 conditions (4) and as a result any physical observable proves to be a function of the generators modulo constraints. So, a general solution to (13) reads

$$A = f(J_{AB}, P_C) + \sum_{i=1}^5 \alpha_i T_i , \quad (14)$$

$\alpha_i$  being arbitrary function of the phase space variables. In fact, this implies that the physical phase space of the model is embedded in the linear space of the Poincaré algebra through the

### 3 Quantization and relativistic harmonic analysis on $CP^3$

As it was argued in previous section, the model is completely characterized at the classical level by the algebra of observables associated with the phase space generators of the Poincaré group, so that any gauge invariant phase space function could be expressed via  $J_{AB}$  and  $P_C$ . From this point of view the quantization of the theory reduces to an explicit construction of irreducible unitary representation of the Poincaré group and may be carried out by means of geometrical quantization method [18].

Within the framework of covariant operatorial quantization, the Hilbert space of physical states of the system is embedded into the space of smooth complex functions on  $M^{12}$  and the phase space variables  $x^A, p_A, \lambda_a, \pi^a$  are considered to be Hermitian operators subject to the canonical commutation relations.

In the ordinary coordinate representation:  $p_A \rightarrow -i\partial_A$ ,  $\pi^a \rightarrow -i\partial^a$ ,  $\bar{\pi}^a \rightarrow -i\bar{\partial}^a$  the quantum first-class constraints take the form

$$\begin{aligned}\hat{T}_1 &= \square - m^2, \quad \hat{T}_2 = \partial^{ab}\bar{\lambda}_a\lambda_b\partial_{cd}\bar{\partial}^c\partial^d + m^2\delta_1^2, \quad \hat{T}_3 = \lambda_a\bar{\lambda}_b\bar{\partial}^a\partial^b - \delta_2^2 \\ \hat{T}_4 &= -i\lambda_a\partial^a, \quad \hat{T}_5 = -i\bar{\lambda}_a\bar{\partial}^a\end{aligned}\tag{15}$$

The subspace of physical states is extracted by the conditions

$$\hat{T}_i |\Phi_{phys}\rangle = 0, \quad i = 1, \dots, 5\tag{16}$$

Notice that the classical dynamics is consistent with arbitrary values of the parameters  $\delta_1, \delta_2$ . Nevertheless, the nontrivial solutions to the equations for physical states (16) can exist only provided that  $\delta_1^2 = s_1(s_1 + 3)$ ,  $\delta_2^2 = s_2(s_2 + 1)$  where  $s_1 \geq s_2$ ,  $s_1, s_2 = 0, 1, 2, \dots$ .

Let us now consider the space  ${}^\uparrow\mathcal{H}(M^{12}, m)$  of massive positive frequency fields on  $M^{12}$ . These fields are annihilated by the constraints  $\hat{T}_1, \hat{T}_4, \hat{T}_5$  and possess the Fourier decomposition

$$\begin{aligned}\Phi(x, \lambda, \bar{\lambda}) &= \int \frac{d\vec{p}}{p_0} e^{i(p, x)} \Phi(p, \lambda, \bar{\lambda}) \\ p^2 + m^2 &= 0, \quad p_0 > 0\end{aligned}\tag{17}$$

The space  ${}^\uparrow\mathcal{H}(M^{12}, m)$  may be endowed with the Poincaré-invariant and positive-definite inner product defined by the rule

$$\langle \Phi_1 | \Phi_2 \rangle = i \int \frac{d\vec{p}}{p_0} \int_{CP^3} \omega \wedge \bar{\omega} \Phi_1 \bar{\Phi}_2\tag{18}$$

where the three-form  $\omega$  is given by

$$\omega = \frac{\epsilon^{abcd}\lambda_a d\lambda_b \wedge d\lambda_c \wedge d\lambda_d}{\left(\bar{\lambda}_a p^{ab} \lambda_b\right)^2}\tag{19}$$

Then  ${}^\uparrow\mathcal{H}(M^{12}, m)$  becomes the Hilbert space and, as a result, the Poincaré group representation is unitary in this space. This representation is decomposed into the direct sum of irreducible ones

where the invariant subspace  ${}^\uparrow\mathcal{H}_{s_1, s_2}(M^{12}, m)$  realizes the Poincaré representation of mass  $m$  and spin  $(s_1, s_2)$  and thereby satisfies all the quantum conditions (16). The explicit expression for an arbitrary field from  ${}^\uparrow\mathcal{H}_{s_1, s_2}(M^{12}, m)$  reads

$$\Phi(p, \lambda, \bar{\lambda}) = \Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}} \frac{\lambda_{a_1} \dots \lambda_{a_{s_1}} \bar{\lambda}_{a_{s_1+1}} \dots \bar{\lambda}_{a_{s_1+s_2}} \bar{\lambda}_{b_1} \dots \bar{\lambda}_{b_{s_1-s_2}}}{(\bar{\lambda}_a p^{ab} \lambda_b)^{s_1}} \quad (21)$$

Here the spin-tensor  $\Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}}$  is considered to be  $p$ -transversal

$$p_{a_1 b_1} \Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}} = 0 \quad (22)$$

(for  $s_1 \neq s_2$ ) and its symmetry properties are described by the following Young tableaux:

$a_1$	$\cdot$	$\cdot$	$a_n$		$\cdot$	$\cdot$	$a_m$	$n = s_1 - s_2$
$b_1$	$\cdot$	$\cdot$	$b_n$					$m = s_1 + s_2$

The field  $\Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}}$  can be identified with the Fourier transform of spin-tensor field on Minkowski space  $R^{5,1}$ . The mass-shell condition

$$(p^2 + m^2) \Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}} = 0 \quad (23)$$

and relation (22) constitute, together, the full set of relativistic wave equations for the mass- $m$ , spin- $(s_1, s_2)$  field in six dimensions. Thus a massive scalar field on  $M^{12}$  generates massive fields of arbitrary integer spins in Minkowski space.

It is instructive to rewrite the inner product for two fields from  ${}^\uparrow\mathcal{H}_{s_1, s_2}(M^{12}, m)$  in terms of spin-tensors  $\Phi(p)^{a_1 \dots a_{s_1+s_2} b_1 \dots b_{s_1-s_2}}$ . The integration over the spinning variables, being performed with the use of the integral  $\int_{CP^3} \omega \wedge \bar{\omega} = -48i\pi^3 (p^2)^{-2}$ , results with

$$\langle \Phi_1 | \Phi_2 \rangle = N \int \frac{d\vec{p}}{p_0} \Phi_1(p)^{a_1 \dots a_{2s_1}} \bar{\Phi}_2(p)_{a_1 \dots a_{2s_1}}, \quad (24)$$

where

$$\bar{\Phi}_2(p)_{a_1 \dots a_m b_1 \dots b_n} = \epsilon_{a_1 b_1 c_1 d_1} \dots \epsilon_{a_n b_n c_n d_n} p_{a_{n+1} c_{n+1}} \dots p_{a_m c_m} \bar{\Phi}_2(p)^{c_1 \dots c_m d_1 \dots d_n} \quad (25)$$

and  $N$  is some normalization constant depending on  $s_1$  and  $s_2$ .

Notice that in the rest reference system  $p_A = (m, 0, \dots, 0)$  the differential operators entering the constraints  $\hat{T}_2, \hat{T}_3$  turn to the conventional  $SO(5)$ -invariant Laplace operators on  $CP^3$  and rel. (20) becomes the standard expansion for a scalar field on  $CP^3$  via the 'spherical' functions. That is why the Poincaré invariant constructions of this Section may be thought of as the *relativistic* harmonic analysis on  $CP^3$ .

The treatment of the half-integer spin representations may be performed along the similar lines by considering the space of special smooth tensor fields on  $M^{12}$  instead of the space of scalar functions  ${}^\uparrow\mathcal{H}(M^{12}, m)$  (cf. see [19]).

## 4 Concluding remarks

In this talk we have suggested the model for a massive spinning particle in the six-dimensional Minkowski space as a mechanical system with configuration space  $M^{12} = R^{5,1} \times CP^3$ . The Lagrangian of the model is unambiguously constructed from the  $M^{12}$  world line invariants when

inconsistent, whereas the homogeneous background is admissible. The physical cause underlying this inconsistency is probably that the local nature of the inhomogeneous field may contradict to the nonlocal behavior of the particle dynamical histories. The possible method to overcome the obstruction to the interaction is to involve the Wess-Zumino like invariants omitted in the action (6). The similar trick solves this problem in the case of  $d = 4$  spinning particle [16].

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